

Technical Note - Modelling Structural Resistance

1 Introduction

LimitState:GEO is able to accurately model bending, shear, tensile and compressive resistances at specific locations in structural elements such as footings, walls or props without having to fully model the whole element. This note explains the underlying principles of this process using simple structural elements as examples. Topics covered in this note are restricted to structural elements that are constructed from **Solids** only. One-dimensional structural elements (**Engineered Elements**) such as soil nails or sheet pile walls are covered separately in the the LimitState:GEO manual Section 6.1.5.

All files used in this note are available in a zip file that can be downloaded from http://www.limitstate.com/files/technical-notes/LSGTN4/modelling_structural_resistance.zip

2 Bending resistance

2.1 Theory

Modelling structural bending resistance in LimitState:GEO utilises the same principles as conventional rigid-plastic structural analysis. Bending failure occurs when a plastic hinge is formed i.e. when the section stresses are all at yield. At the ultimate state rectangular stress blocks are assumed as shown in Figure 1. The full plastic moment can be calculated using equation 1.

$$M_p = \sigma_y \frac{bd^2}{4} \quad (1)$$

where d is the section depth, b the section breadth, σ_y is the section limiting yield stress and M_p is the full plastic moment of resistance of the section i.e. section bending resistance.

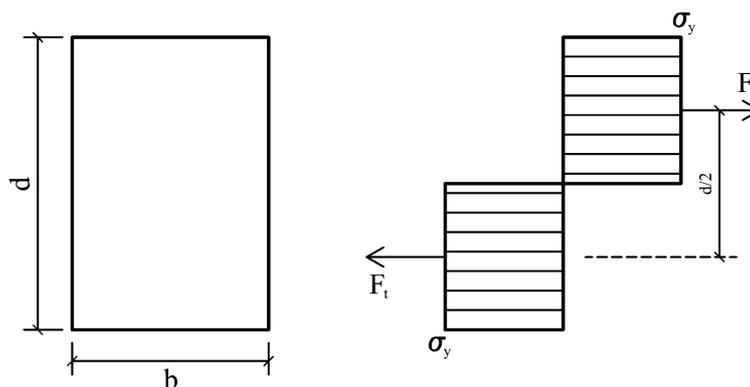


Figure 1: Stress blocks at the ultimate limit state for a pure bending condition

LimitState:GEO allows for structural bending failure to occur only at a pre-specified deformable interface. It is required therefore to insert a deformable interface anywhere where a hinge is expected to form. The yield surface itself is to be modelled with a **Cutoff** material. This is defined in terms of a limiting tensile stress σ_T and limiting compressive stress σ_C as depicted in Figure 2. When modelling bending with a **Cutoff** material the specified limiting compressive and tensile stresses are both equal to the yielding stress σ_y in equation 1 i.e. $\sigma_T = \sigma_C = \sigma_y$.

Since the analysis in LimitState:GEO is plane strain, equation 1 can be simplified to the form in equation 2 used when modelling bending resistance with LimitState:GEO:

$$\sigma_T = \sigma_C = \sigma_y = \frac{4M_p}{d^2} \tag{2}$$

where M_p is now the full plastic moment of the section per unit width.

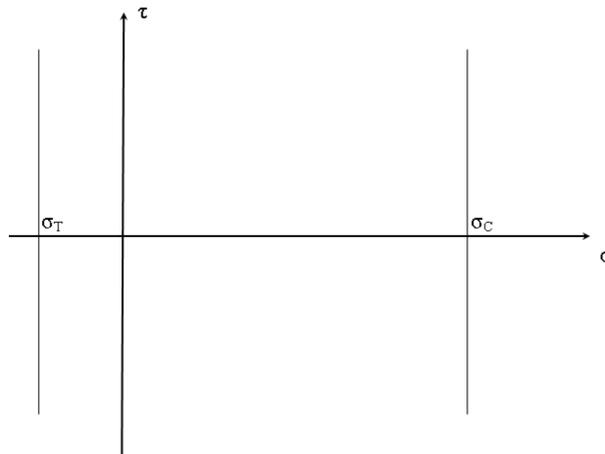


Figure 2: Cutoff material properties

2.2 Example - bending

The concept of modelling plastic bending in LimitState:GEO discussed in Section 2.1 will be explained using the example of a simply supported beam subjected to a uniformly distributed load.

The problem models a weightless 0.4 m deep plastic beam supported on two rigid blocks with a uniformly distributed load of 20 kN/m² applied to the top of the beam as shown in Figure 3. The clear length of the beam is 4.0 m and the sectional plastic moment of resistance is $M_p = 40$ kNm/m.

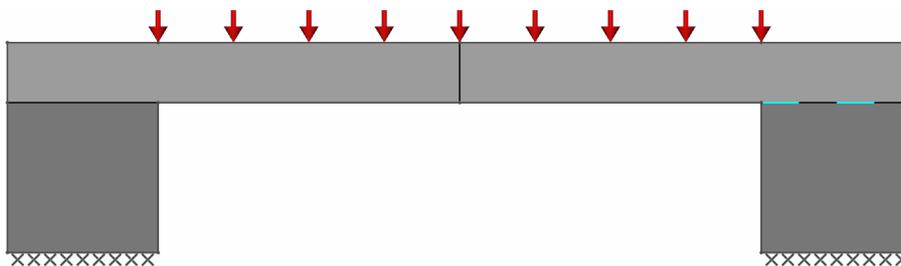


Figure 3: Plastic beam model

The beam/block interfaces are modelled with no-tension cutoff to allow them to rotate about the inner corners and one is also modelled smooth to avoid inducing horizontal loads (hence effectively modelling a roller bearing). The beam itself is modelled as two rigid blocks connected by a vertical boundary at the centre of the beam where formation of a plastic hinge is expected. On this boundary a yield surface using a **Cutoff** material allowing tensile and/or compressive yield to occur at specified stresses. Properties of the yielding surface are listed in Table 1.

The model as defined above is available as [plastic_beam.geo](#)

| Material type | Limiting compressive stress (kN/m ²) | Limiting tensile stress (kN/m ²) | Thickness (m) | Equivalent bending resistance (kN.m/m) |
|---------------|---|---|------------------|---|
| Cutoff | 1000 | 1000 | 0.4 | 40 |

Table 1: Yielding surface properties

- 1) Open the model
- 2) Click **Solve** or  This returns an **Adequacy factor** of 1.0 matching the benchmark with 0% discrepancy.
- 3) Check the stress blocks on the yielding surface by clicking on the one of the beam elements. Hovering the arrow pointer over the stress block causes display of the numerical values of the average normal and shear stresses.

NB. LimitState:GEO requires a reasonable number of nodes to be present on the yielding interface to create accurate stress blocks. In principle pure bending can be achieved with only one additional node at the middle of the interface. If however significant axial forces are acting on the element, then the neutral axis will be displaced from the interface centreline and it should be checked whether there are sufficient number of nodes on the yielding interface to represent this position.

2.3 Example - bending and axial force

The plastic beam model is now altered by adding compressive axial pressure of 500 kN/m² at both ends of the beam. In addition, the number of nodes on the yielding interface has been decreased.

The theoretical reduced value of the plastic moment M_a is given by equation 3.

$$\frac{M_a}{M_p} = 1 - \left(\frac{P}{P_y} \right)^2 \quad (3)$$

where P is the axial force and P_y is the axial force required to cause yield in pure tension/compression.

In this example $M_a = M_p(1 - (500/1000)^2) = 0.75M_p$.

- 1) Open the [plastic_beam_with_axial_forces.geo](#) model.
- 2) Click **Solve** or . An **Adequacy factor** of 0.7778 is returned.
- 3) Check the stress blocks at the yielding interface by clicking on the one of the beam elements. The shape of the stress blocks and small number of nodes on the boundary indicates that result might be inaccurate. (The software will attempt to approximate the true distribution by not modelling full yield on the blocks in the vicinity of the neutral axis as shown in Figure 4a).
- 4) Unlock the model using .
- 5) Increase the number of nodes at the yielding interface. Click the appropriate boundary and in the **Property Editor** decrease the **Baseline Nodal Spacing** to 0.5.
- 6) Click **Solve** or . The returned **Adequacy factor** is now 0.75.
- 7) Check the stress blocks at the yielding surface by clicking on the one of the beam elements (Figure 4b). The nodal density is clearly sufficient now and any further increase in number of nodes at the yielding surface will not have any influence on the result.

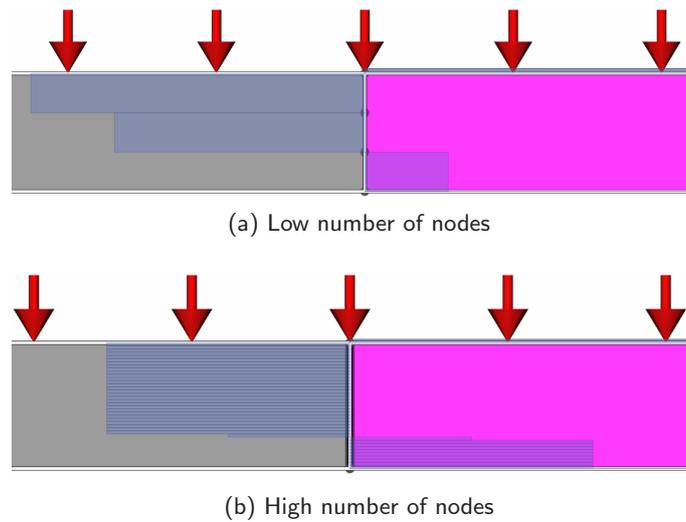


Figure 4: Stress blocks for beam bending with axial force present

3 Tensile and compressive resistance

Modelling compressive/tensile resistance is of interest for axially loaded structural elements such as struts or props. The suggested approach is very similar to the one used for modelling bending explained above. The structural element itself is to be modelled as a rigid body with a specified deformable interface represented by a **Cutoff** material. The specified deformable interface has to be normal to the main axis of the element. This is because the **Cutoff** material limits only normal stress as depicted in Figure 2. The limiting tensile stress σ_T and the limiting compressive stress σ_C represent the tensile and compressive strength of the element respectively. Note that when specifying σ_C , the minimum strength of the element should be always considered e.g. the buckling resistance in compression.

The example prop model with properties as listed in Table 2 is available as [prop.geo](#) and is shown in Figure 5. The applied load is $1000 \times 0.2 = 200$ kN/m. On solve an **Adequacy factor** of 1.0 is returned indicating yield at the expected axial load.

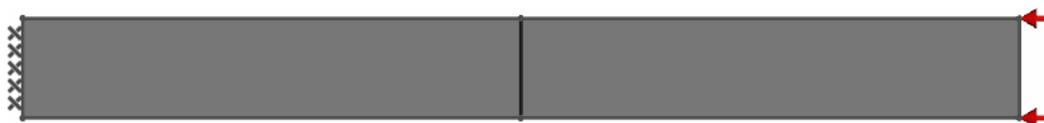


Figure 5: Prop model

| Material type | Limiting compressive stress (kN/m ²) | Limiting tensile stress (kN/m ²) | Width (m) | Equivalent compressive resistance (kN/m) |
|---------------|--|--|-----------|--|
| Cutoff | 1000 | Not relevant | 0.2 | 200 |

Table 2: Deformable interface properties

4 Shear resistance

When modelling structural shear resistance in LimitState:GEO, a **Mohr-Coulomb** material may be used with cohesion c_u or c' representing the shear strength of the structural element. The element itself can be modelled

in two ways:

- 1) As a solid made entirely of the **Mohr-Coulomb** material.
- 2) As a rigid body with specified shearing interfaces modelled with a **Mohr-Coulomb** material.

Where possible, the second approach is recommended. This is compatible with the way of modelling bending resistance in LimitState:GEO and is more computationally efficient. If the first approach is to be used the user should be aware that the specified cohesion for the structural element will determine its overall resistance. It means that the shear strength will be properly represented only if shearing is the critical failure mode within the element. Otherwise different incorrect failure modes might be returned by the LimitState:GEO.

An example of modelling shear resistance of a cantilever beam in LimitState:GEO is available as [cantilever_shear.geo](#). The properties of the shearing surface (used at the junction of the beam and column) are listed in Table 3 (unit weights γ and γ_{sat} are not relevant for materials used only on boundaries) and the applied distributed load is 40 kN/m².

The applied load is $40 \times 2 = 80$ kN/m. On solve an **Adequacy factor** of 2.5 is returned indicating yield in shearing at the expected shear load.

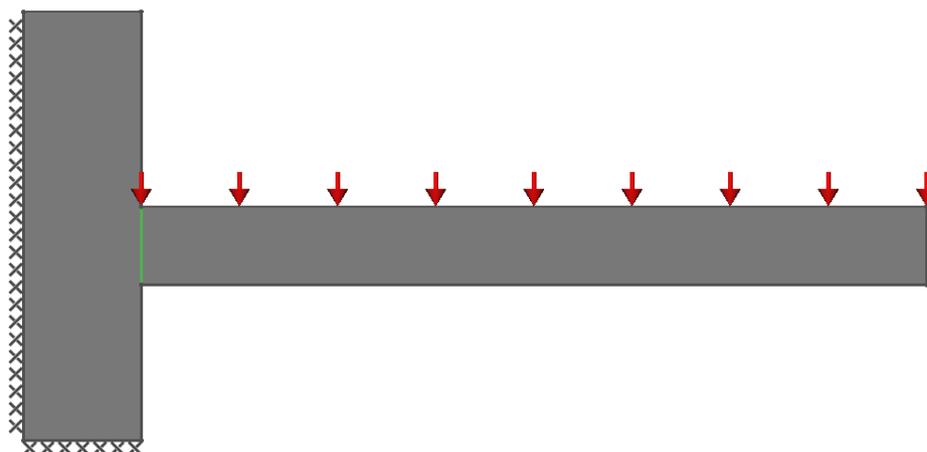


Figure 6: Cantilever model

| Material type | Material drainage behaviour | c_u (kN/m ²) | c' (kN/m ²) | ϕ' | Length (m) | Equivalent shear resistance (kN) |
|---------------|-----------------------------|-------------------------------|------------------------------|---------|---------------|-------------------------------------|
| Mohr-Coulomb | Always drained | - | 1000 | 0 | 0.2 | 200 |

Table 3: Shearing surface properties

5 Combined shear and bending resistance

Different types of resistance can be modelled with one boundary using a **combined material**. To create a **combined material** drag and drop materials representing each type of resistance onto the boundary. Whilst dropping the second one, the **combined material** dialog will appear. Select the **Add** option.

For example bending and shear can be simultaneously modelled in the cantilever beam example. If the cutoff material defined in Table 1 (this new material will need to be created) is added to the beam/column interface (resulting in a plastic moment of bending resistance of 10 kNm/m), then the beam fails in bending and a revised

Adequacy factor of 0.125 is returned. This generates the expected failure bending moment at the beam/column interface ($0.125 \times 40 \times 2 \times 1 = 10 \text{ kNm/m}$)

Note that it is not possible to model bending and tensile/compressive resistance on the same boundary.

For more information: www.limitstate.com/geo