

Technical Note - The Influence of Arch Shape on Masonry Arch Bridge Capacity

1 Introduction

It is sometimes forgotten that it is the shape of an arch in relation to the pattern of applied loading which governs stability. Hence it is of paramount importance that due care is taken when recording and entering the shape of an arch barrel into any analysis program. Indeed it can be considered just as important to accurately record the arch shape and thickness as it would, for example, to accurately measure the overall depth, flange thickness etc. of a steel beam prior to assessing its ability to carry a given load.

Furthermore, when transverse cracks are evident through the barrel, it follows that the arch profile must differ from that originally constructed, and this makes it even more important to perform an accurate survey of the bridge prior to analysis, to ensure that the true shape of the arch is modelled.

This feature discusses why it is necessary to accurately model the arch shape correctly when performing an assessment and considers the different profile types available in LimitState:RING that allow users to achieve this.

2 Some lessons from history

In 1675 the English scientist Robert Hooke (1635 - 1703) presented his solution to the longstanding masonry arch stability problem in the form of a Latin anagram in an appendix to his work 'Description of Helioscopes'. Although the solution to the puzzle was not revealed during his lifetime, it was eventually established to read:

"ut pendet continuum exile, sic stabit contiguum rigidum inversum"

which roughly translates to:

"as hangs a flexible cable, so but inverted will stand the rigid arch"

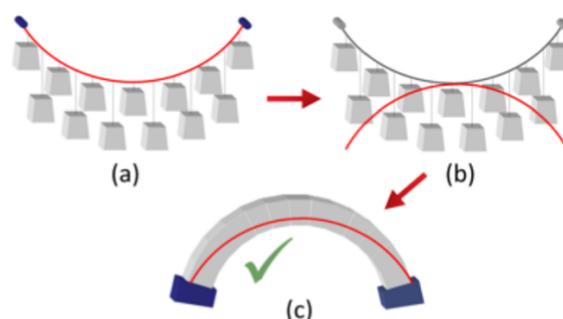


Figure 1: Hooke's theory of the inverted cable: a) masonry blocks hanging on a weightless cable; b) the profile of the cable inverted; c) the inverted profile fits within the thickness of the masonry blocks, so demonstrating that the arch can stand

In practical terms, what Hooke meant was that if the blocks to be used to form an arch are hung from a weightless cable (Figure 1a), and the profile of the cable is then inverted (Figure 1b), as long as the latter profile fits entirely within the thickness of the masonry then the arch will stand (Figure 1c). Whilst the hanging chain is in tension, the inverted line represents a feasible line of compression, or thrust, in the masonry arch.

Hooke's theory certainly ought to have proved helpful to William Edwards, constructing a bridge some decades later at Pontypridd, but communication channels were at that time not well developed, and it appears that Edwards was unaware of Hooke's theory.

Following the destruction of his original three span arch bridge during a heavy storm in 1746, Edwards designed a radical 140ft replacement that would stretch across the river Taff in a single span. After an initial unsuccessful attempt, the bridge was eventually completed. However, after the centering was removed it stood for only six weeks, the failure being attributed to too much weight being applied at the haunches in comparison to that applied at the crown. This led to the line of thrust migrating outside the thickness of the masonry barrel, eventually leading to collapse of the bridge. Unperturbed, Williams set about redesigning the bridge - this time introducing voids in the spandrel zones to reduce the weight over the haunches and therefore altering the profile of the line of thrust. The resulting structure was successful, and still stands today, several hundred years later (Figure ??).



Figure 2: Williams' bridge at Pontypridd, showing spandrel voids

3 Modelling arch shape in LimitState:RING

From the above discussion it is clear that the shape and thickness of the arch barrel should if possible be well defined when carrying out any assessment of bridge load carrying capacity. However, sometimes the default 'segmental' arch shape in LimitState:RING is used when it doesn't accurately represent the true shape of the arch. Whilst this is to some extent unavoidable when geometrical data is lacking, when such data becomes available the true shape of an arch can be faithfully modelled in LimitState:RING, using the various modelling options available:

- **Segmental** *The arch profile is formed from a single segment of a circle, constructed using the crown rise and span measurements.*
- **User-defined (multi-segment)** *The arch profile is formed from multiple segments of circles which fit a series of user-defined data points.*

The following modelling options are new in LimitState:RING and together provide a significantly enhanced capability:

- **User-defined (interpolated)** The arch profile is formed from a spline interpolation of user defined data points. (This is the most powerful and flexible profile type, suitable for use when many user-defined data points are specified.)
- **Three-centered (pseudo-elliptic)** The arch profile is (near) elliptical in shape, being formed from segments of three circles using the crown rise and span measurements.
- **Pointed** The arch profile is pointed and is formed from segments of two circles using the quarterspan rise, crown rise and span measurements. This can therefore represent a 'gothic' arch.

4 Example - how arch shape affects capacity

To illustrate how the modelled shape can influence assessed capacity, consider a 5m span bridge with a 4 to 1 span to rise ratio and 750mm fill over the crown. When a segmental profile is used only the span and midspan rise are needed to define the arch shape. Assuming a single axle load to be critical, a benchmark load carrying capacity can be established. However, now suppose the quarterspan rises are actually different to that implicitly assumed in the segmental profile (but that the shape remains smooth). Figure ?? shows the reduction in capacity for a series of such cases, calculated using the 'User-defined (interpolated)' arch profile and 9 data points:

- **Case 1** - both quarterspans lower than in the segmental profile.
- **Case 2** - both quarterspans higher than in the segmental profile.
- **Case 3** - An asymmetrical arch with one quarterspan higher and one lower (by the same amount) than in the segmental profile.

It can be seen that the capacity of the arch quickly reduces for even a small percentage difference in quarterspan rise. Furthermore, this reduction is most marked for an asymmetrical arch shape, which can occur as a result of overloading and is therefore of particular significance. This highlights the necessity to ensure that the arch shape is modelled as accurately as possible when assessing a bridge.

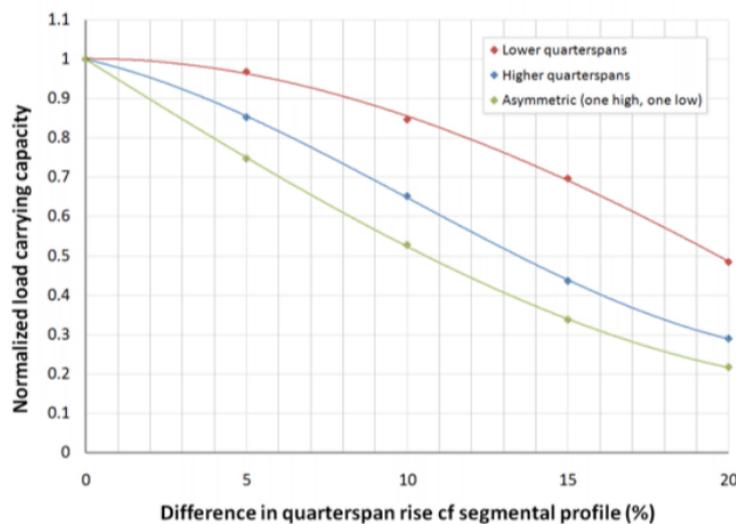


Figure 3: Graph of normalized load carrying capacity vs quarterspan rise deviation when compared to that of a benchmark segmental arch bridge.

5 A word of caution

Whilst the available profiles in LimitState:RING allow the engineer to model a wide variety of different arch shapes, the engineer must always ensure that the resultant profile is credible. For instance, arch barrels with a 'widow's peak' (Figure 4a) or 'bell' (Figure 4b) shaped profile can be produced in some extreme situations, but these are very likely not to exist in reality and would lead to significant underestimates of the bridge capacity if left unresolved. The advice is therefore to always check the profile in the model against photographs and / or drawings of the bridge itself, making any alterations that are required before finalising an assessment.

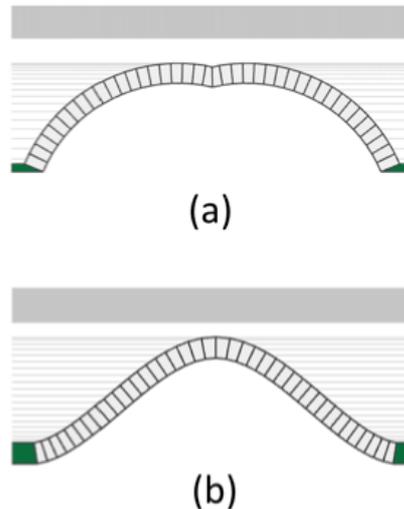


Figure 4: Non-credible arch profiles: a) 'Widow's peak' and b) 'Bell' shape.

For more information: www.limitstate.com/ring