Selected Publications

LimitState:RING uses the rigid-block analysis procedure and was originated and developed by LimitState co-founder Dr Matthew Gilbert. Presented here is selected list of publications related to LimitState:RING, the rigid-block methodology and its application; if you would like to obtain a copy of any of the publications mentioned please contact info@limitstate.com.

Rigid-block analysis of masonry structures

This paper describes a conceptually simple and computationally efficient method of determining the collapse load of structures comprising of a number of masonry blocks. The method uses the upper-bound theory of plasticity in conjunction with geometrical compatibility criteria to obtain solutions to problems involving single- and multi-span arches; well established rigorous linear programming methods are used to obtain solutions. Specific parameters such as ring separation and attached spandrel walls can be modelled using the method. It is expected that case studies will be described in subsequent papers.

The behaviour of multiring brickwork arch bridges

A considerable amount of work has in recent years been devoted to the study of the behaviour of single-span masonry arch bridges. Despite the large numbers of multispans bridges in this country and abroad, much less attention has been afforded them. This paper describes a programme of tests on three large-scale model multispans brickwork arch bridges, constructed in the laboratory. In the case of each bridge, when tested to collapse, it was found that one or more of the spans adjacent to the loaded span were involved in the failure mechanism. It was also found that the critical loading position was not at quarter span, but closer to the crown. The presence of attached spandrel walls had a significant influence on the behaviour and strength of the bridges. The experimental results are found to compare reasonably well with theoretical predictions using a rigid block (mechanism) method of analysis, especially when spandrel walls are included in the calculations.

The collapse behaviour of multispans brickwork arch bridges

A large amount of work has in recent years been devoted to the study of the collapse load of structures comprising of a number of masonry blocks; this is because the constituent rings will often separate, causing premature failure of the bridge. Most analysis techniques developed to date make the implicit assumption that shear failure between rings will not occur; it is demonstrated that these methods are inappropriate for the calculation of the strength of multiring brickwork arch bridges. This paper describes a series of tests carried out on multiring brickwork arch bridges tested in the laboratory. The tests showed that significant reductions in strength accompany ring separation. The ‘rigid block’ mechanism method of analysis, described in an earlier paper, has been used to determine the ultimate carrying capacities and failure modes these bridges. The collapse loads of the bridges built with the defect of ring separation were often predicted with a reasonable degree of accuracy; the collapse loads of bridges built with mortar bonded arch barrels were more difficult to quantify, because of the unpredictability of brittle structure of the mortar bonding together the arch rings.

On the analysis of multi-ring brickwork arch bridges

As adjacent rings of brickwork in multi-ring brickwork arch bridges are adhered by mortar joints that are relatively weak in tension and shear, it is frequently observed that these become debonded, resulting in delamination or “ring separation”. A rigid block method of analysis has previously been used to calculate the collapse loads of multi-ring brickwork arch bridges that have very weakly mortared arch rings. This method of analysis is extended in the paper to allow the effect on carrying capacity of crushing of the masonry at hinge positions to be estimated. Significantly, it is predicted that the difference between the carrying capacity of a voussoir arch and the carrying capacity of a multi-ring arch reduces as the compressive strength of the masonry is reduced. Other practical issues associated with the analysis assessment of bridges of this type are also discussed in the paper.

RING: A 2D rigid block analysis program for masonry arch bridges

A rigid block analysis formulation for masonry arches was first put forward by Livesley more than two decades ago. However, despite the advantages of the method, few researchers or practitioners have had access to software based on the rigid block analysis formulation. With a view to remedying this, rigid block analysis software developed by the author for personal research use has recently been re-assessed. It was found that in addition to cosmetic changes (required to improve the user-interface), several more significant developments were required in order to make the tool more robust and easy to use, and also to ensure that the software had reasonably wide applicability. For example, it was found that the addition of uniaxial fill elements in the analysis obviated the need for user intervention when dealing with certain problems involving horizontal backfill pressures, and also that these elements did not change the linear nature of the problem. The resulting software has been named RING, and is now freely available via the web.
The computational efficiency of two rigid block analysis formulations for application to masonry structures


Rigid block analysis is a computational limit analysis method which is now widely applied to the analysis of masonry gravity structures such as arch bridges. Various formulations of the rigid block analysis method have been proposed, with linear programming (LP) generally being used in the solution process. However, there appears to be little information in the literature on the relative computational efficiencies of the various formulations (when used either with traditional simplex or newer interior point LP solvers). Thus a “redundant forces” formulation put forward previously by the second author is here compared with an alternative “joint equilibrium” formulation. It is found that the joint equilibrium formulation is the most computationally efficient formulation when applied to complex geometrical arrangements of blocks, such as those found in multi-ring arches.

Developments to the RING masonry arch bridge analysis software


The RING computational limit analysis software for masonry arch bridges was originally developed as a research tool in the early 1990’s but has been publicly available via an internet site since 2001. This paper describes recent and planned future developments to the software which is currently being completely rewritten, including: order of magnitude improvements in its computational efficiency; a non-associative constitutive model for sliding friction; enhancements to the general usability of the software. The new software is being developed using an object-oriented approach so that further significant developments (e.g. 3D arch analysis and full coupled arch-soil limit analysis) should be able to be readily incorporated in the future.

Load-carrying capacity of flooded masonry arch bridges


Many existing codes of practice for masonry arch bridge assessment fail to provide engineers with adequate guidance on the adverse effects that saturated environmental or flood conditions are likely to have on load-carrying capacity. Analysis indicates that the load-carrying capacity of a fully flooded arch bridge backfilled with cohesionless fill could typically be reduced by a factor of 1.6–1.8, or even more in specific circumstances. The significant influence of flooding on load-carrying capacity has now been verified using small-scale experiments which are described in the paper. The effects of flooding on waterproofed and unwaterproofed bridges are also analysed numerically and the circumstances under which capacity reductions are likely to be highest are identified. This paper should be useful to engineers dealing with bridges which are at risk of flooding.

Limit analysis applied to masonry arch bridges: state-of-the-art and recent developments


Limit analysis potentially provides a highly effective means of verifying the safety of structures and has successfully been applied to masonry arch bridges for many years. Hand based limit analysis techniques have been largely superseded by computer based methods which are the primary focus of this paper. Recent developments to ‘thrust line’, discrete ‘rigid block’ and various combined soil-arch interaction limit analysis models for masonry arch bridges are discussed and areas where further work is required are identified.

Small and large-scale experimental studies of soil-arch interaction in masonry bridges


The interaction between the arch, its backfill and any applied surface load is complex, and only a limited amount of relevant experimental research has been undertaken to date. Small-scale tests, which are inexpensive and quick to perform, have recently been used to try to separate the effects of load-spreading and passive restraint, whilst full-scale model bridges (3m span and housed in a large, 8.3 m long x 2.1 m high and extremely stiff test chamber), have been used to furnish high quality test data which can be correlated against numerical models, being developed in parallel. Both small and full-scale bridges are being tested in chambers incorporating large observation windows along one face to permit digital imaging and quantitative measurements of the soil and arch movements, allowing subsequent correlation with numerical models. Key results are presented, including those from full-scale model bridges filled with crushed limestone and/or soft clay.